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On Grothendieck spaces $C(X)$ and complemented copies of $(c_0)_p$ in spaces $C_p(X \times Y)$ and $C_p(X, E)$

It is known that the Banach space $C(X \times Y)$ always contains a complemented copy of the Banach space c_0 for infinite compact spaces X and Y (Cembranos-Freniche). The aim of the talk is to summarize several (older and very recent) results, concepts and ideas concerning the corresponding results for the spaces $C_p(X \times Y)$ of continuous functions on $X \times Y$ endowed with the pointwise topology. For example, one shows a theorem (implying also Cembranos-Freniche result) stating that for all infinite Tychonoff spaces X and Y the space $C_p(X \times Y)$ contains either a complemented copy of $\mathbb{R}^\omega$ or a complemented copy of the space $(c_0)_p = \{(x_n)_{n\in\omega} \in \mathbb{R}^\omega \colon x_n \to 0\}$, both endowed with the product topology. On the other hand, assuming the Continuum Hypothesis, there are examples of pseudocompact spaces X such that $C_p(X \times X)$ does not contain a complemented copy of $(c_0)_p$. This approach provides new concepts related with the Grothendieck spaces $C(K)$ and $C(K, E)$ over compact spaces K. Several applications will be provided.