Vector lattices of real-valued functions that are stable under reciprocation

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Abstract

Let $\mathfrak{V}$ be a vector lattice of real-valued functions defined on a nonempty set containing the constant functions. It is shown that if the reciprocal of each non-vanishing member of $\mathfrak{V}$ is again in $\mathfrak{V}$, then $\mathfrak{V}$ is a ring under pointwise product. Our interest is in vector lattices of functions defined on a metric space. Strangely, neither of the two classes of metric spaces for which these properties hold in the case of the vector lattice of uniformly continuous function was adequately described until after 2015! We outline the relevant results relative to this particular vector lattice of functions and identify those metric spaces on which the real-valued Lipschitz functions are stable under reciprocation (resp. pointwise product). We then turn our attention to certain classes of Lipschitz-like functions. Results under discussion reflect joint work with Maribel Garrido, Ana Meroño, and Luis Carlos Lirola-García published in Set-Valued and Variational Analysis and RACSAM. As to when the uniformly continuous real-valued form a ring, the key results are due to Javier Cabello-Sanchez and separately Ahmed Bouziad with Elena Sukhacheva.