Δ-spaces X and distinguished spaces $C_p(X)$

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Definition 1. *A subset of reals $X \subset \mathbb{R}$ is said to be a* <u>Δ-set</u> *if X has the following* <u>property Δ</u>: *for every decreasing sequence $\{D_n : n \in \omega\}$ of subsets of X with empty intersection, there is a decreasing sequence $\{V_n : n \in \omega\}$ consisting of open subsets of X, also with empty intersection, and such that $D_n \subset V_n$ for every $n \in \omega$.*

The existence of an uncountable Δ-set $X \subset \mathbb{R}$ is one of the fundamental set-theoretical problems; it is independent of ZFC (due to results of F. Hausdorff and M.E. Rudin).

Definition 2. *A locally convex space (lcs) E is called* <u>distinguished</u> *if the strong dual of E (i.e. the topological dual of E endowed with the strong topology) is barrelled* (A. Grothendieck).

<u>What two defined above notions from two completely different areas, one from Set Theory, another from Functional Analysis, can have in common</u>?

By $C_p(X)$ we mean the space of all real-valued continuous functions on a Tychonoff topological space X endowed with the topology of pointwise convergence.

Theorem 3. [1] *Let X be a Tychonoff space. A lcs $C_p(X)$ is distinguished if and only if for each $f \in \mathbb{R}^X$ there is a bounded $B \subset C_p(X)$ such that f belongs to the closure of B in $\mathbb{R}^X$.*

We say that a Tychonoff space X is a <u>Δ-space</u> if X satisfies property Δ, as in Definition 1 above. Quite recently we have shown that the notion of Δ-spaces plays a key role in the study of distinguished C_p-spaces [2].

Characterization Theorem 4. [2] *Let X be a Tychonoff space. A lcs $C_p(X)$ is distinguished if and only if X is a Δ-space.*

My talk will be devoted to the main results published in the joint papers [1], [2], [3].

References

[1] J. C. Ferrando, J. Kąkol, A. Leiderman, S. A. Saxon, *Distinguished $C_p(X)$ spaces*, RACSAM **115:27** (2021).

[2] J. Kąkol, A. Leiderman, *A characterization of X for which spaces $C_p(X)$ are distinguished and its applications*, Proc. Amer. Math. Soc., series B, **8** (2021), 86–99.

[3] J. Kąkol, A. Leiderman, *Basic properties of X for which the space $C_p(X)$ is distinguished*, Proc. Amer. Math. Soc., series B, **8** (2021), 267–280.