

Lipschitz functions with values on the unit circle group

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Abstract

If (X, d) is a metric space and 0 is a fixed element of X , $\text{Lip}_0(X, \mathbb{R})$ denotes the Banach space of real-valued Lipschitz functions f defined on X and such that $f(0) = 0$. One of its main properties is that it is isometric to the dual space of the norm-closed subspace of $(\text{Lip}_0(X, \mathbb{R}))^*$ generated by the Dirac measures. The latter space is of interest in its own right: It is the universal Banach space that contains an isometric copy of X .

Motivated by these important concepts we introduce the group of Lipschitz functions $\text{Lip}_0(X, \mathbb{T})$ with values on the unit circle group \mathbb{T} , and the free abelian group $A_d(X)$ on a metric space (X, d) endowed with the topology generated by the Graev extension \hat{d} of the given metric d on X . We will explore the close relation between them and derive some interesting properties of both metric groups, $\text{Lip}_0(X, \mathbb{T})$ and $A_d(X)$.