# KREIN PROPERTY FOR ABELIAN TOPOLOGICAL GROUPS

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ABSTRACT. The consideration of topological vector spaces as abelian topological groups has a long tradition. Already in the 50's M. F. Smith published a paper in which the Pontryagin duality Theorem was considered for the special class of real topological vector spaces.

Along these years many well-known results of Functional Analysis have been reinterpreted for the class of abelian topological groups. With this point of view, in the present talk we mainly deal with a Theorem of Krein and its obstructions for a parallell version in the context of topological groups.

A few words about Mark Gregorievich Krein can be considered as a token of gratitude for his huge production in high quality Mathematics. He was born in Kiev in 1907. In the 30's he created at Odessa a strong center for Functional Analysis, which lasted only until he was dismissed for non scientific reasons. Besides a brilliant mathematician, he was a magnificent pedagogue. According to Mathematical Reviews he has 49 students and 1.005 descendents. This illustrates the impact of his work.

In this talk we focus on the Krein's Theorem which can be stated as: "In a quasi-complete locally convex space $E$, the closed absolutely convex envelope of a weakly compact subset is again weakly compact". A little earlier Mazur had proved a similar statement for Banach spaces with their usual norm topology.

For a topological abelian group $G$, we define: $G$ has the *Krein property* if the quasi-convex hull of every weakly compact subset of $G$ is again weakly compact. In the context of topological groups the "weak topology" is the initial topology for the family of all continuous characters. We shall present a family of complete metrizable locally quasi-convex groups which do not have the Krein property and analyze related features.

The talk is based on joint research with T. Borsich, X. Domínguez and V. Tarieladze.